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New phases in CFL quark matter

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We consider $O(\alpha_s)$ corrections to the squared masses of the pseudo-Goldstone excitations about the ground state of dense quark matter. We show that these contributions tend to destabilize the vacuum, leading to a surprisingly complex phase structure for quark matter as a function of quark mass, even for small α_s. In particular we find two new phases of CFL quark matter possibly relevant for the real world, for which $\bar{\theta}_{\rm QCD} = \pi/2$.

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The dependence of the QCD vacuum on the masses of the light quarks is most efficiently analyzed by means of a chiral Lagrangian, which allows one to systematically compute corrections to the chirally symmetric vacuum in a power expansion in the quark masses. The utility of the chiral Lagrangian approach is that it properly accounts for the lightest QCD excitations, the pseudo Goldstone bosons (PGBs), whose masses vanish in the chiral limit and which therefore have the most important role in determining the vacuum structure for small quark mass. In addition to allowing one to compute perturbative corrections to the vacuum structure, the chiral Lagrangian also allows one to investigate phase transitions at critical values of the quark masses. An example of such a phase transition was first discovered by Dashen [1], who found a line of phase transitions in the (m_u, m_d) plane corresponding to spontaneously broken CP symmetry, where $m_{u,d}$ are the up and down quark masses respectively.

In hadronic matter at nonzero baryon density, more complicated phase transitions have been investigated using chiral Lagrangians, including both pion [2] and kaon condensation [3, 4]. More recently, chiral Lagrangians have been used to analyze the vacuum structure of dense quark matter near the chiral limit. It has been convincingly argued that for degenerate quarks such a system should be color superconducting [5], and that for three massless flavors the color-flavor-locked (CFL) ground state is preferred [6, 7, 8]. Chiral perturbation theory can be used to describe the ground state and excitations of CFL matter away from the chiral limit [9, 10, 11]. It is already known that the phase structure is much more complicated than in the vacuum case. In particular, an analysis to leading order in both quark masses and the QCD coupling constant α_s reveals a phase transition to a kaon condensed phase along a line in the m_s-m plane [12, 13, 14], where m_s is the strange quark mass, and for simplicity we are considering the isospin limit, $m_u = m_d = m$. In this Letter we show that the leading order calculation does not capture the full complexity of the quark matter ground state, and that there exist new phases revealed at sub-leading order in α_s at small quark mass in the m_s-m plane which could be relevant for the real world.

The reason that $O(\alpha_s)$ effects are important is that the leading order result exhibits an accidental degeneracy which is only lifted at next-to-leading order. Once $O(\alpha_s)$ effects are included the meson masses receive negative corrections to their squared masses which can be larger than the leading order contributions, even for small α_s. In particular, a meson of mass M with $M^2 \propto m^2$ at leading order can become destabilized by a perturbative correction of the form $\delta M^2 \propto -\alpha_s m_s^2$, given that $m_s^2 \gg m^2$. Characteristic of these new phases, which can coexist with kaon condensation, is that they break an exact Z_2 symmetry (parity) and that $\det U \neq 1$, where U is the unitary matrix characterizing the quark condensate. Such phases appear to have a non-vanishing strong CP violating angle, $\bar{\theta}_{QCD} = \pi/2$.

In the CFL phase, attractive interactions near the (shared) Fermi surface cause quarks of all three flavors to undergo BCS-like pairing. The resulting condensate takes the form

$$\langle \psi_i^a C\gamma_5 \psi_j^b \rangle \propto \Delta_3\, \epsilon^{abx}\epsilon_{ijx} + \Delta_6\, \left(\delta_i^a\delta_j^b + \delta_j^a\delta_i^b\right), \qquad (1)$$

where $\Delta_{3,6}$ are the pairing gaps in the antisymmetric color triplet and symmetric color sextet channels respectively. A perturbative calculation yields [7]

$$\Delta_6^2 = \alpha_s \frac{(\ln 2)^2}{162\pi}\Delta_3^2\ . \qquad (2)$$

The gap Δ_3 is larger than Δ_6 because one-gluon exchange is attractive in the triplet channel but repulsive in the

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sextet channel; the radiative generation of nonzero Δ_6 does not affect the symmetries of the CFL phase. The condensate spontaneously breaks the $U(1)_B$ baryon number symmetry, as well as the $SU(3)_c \times SU(3)_L \times SU(3)_R$ symmetry down to the diagonal subgroup, $SU(3)_{c+L+R}$ where $SU(3)_c$ refers to the color gauge group. As a result the gluons become massive, and there appears a nonet of Goldstone bosons analogous to the pseudoscalar meson nonet in the vacuum (in addition to the superfluid mode from the breaking of $U(1)_B$). Unlike in the vacuum, instantons are suppressed at high density, and the entire nonet is expected to be light.

In the CFL phase low energy excitations may therefore be parametrized by $\Sigma = e^{2i\pi/f_\pi}$, $V = e^{4i\eta'/f_{\eta'}}$, and $B = e^{i\beta/f_B}$ with $\pi = \pi^a T^a$, the T^a being $SU(3)$ generators. The π_a and η' form the pseudo-scalar nonet, while β (which we shall ignore for the remainder of this Letter) is the Goldstone boson of broken baryon number. Under the original symmetry of $SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A$ the Σ field transforms as $(3,\bar{3})_{(0,0)}$, V transforms as $(1,1)_{(0,4)}$, where we have assigned axial charges of $+1$ and -1 to the right- and left-handed quarks respectively. A gauge invariant order parameter for the system is (schematically) $\langle \bar{q}_L \bar{q}_L q_R q_R \rangle$ whose orientation in $SU(3)_L \times SU(3)_R \times U(1)_A / SU(3)_V$ is given by the $U(3)$ matrix $U = V\Sigma$. The quark mass matrix $M = \mathrm{diag}(m, m, m_s)$ explicitly breaks the chiral symmetries of QCD, and enters the effective theory as a spurion transforming as $(3,\bar{3})_{(0,-2)} \oplus (\bar{3},3)_{(0,2)}$.

The chiral Lagrangian introduced in [9] and elaborated in [10, 11, 13] is an expansion in quark mass, derivatives, and strong coupling α_s. Generically it takes the form

$$\mathcal{L} \sim \mu^2 \Delta^2 \, \hat{\mathcal{L}} \left(\frac{\partial^2}{\Delta^2}, \frac{M^2}{\mu\Delta}, \frac{M^2}{\mu^2}, \alpha_s, \frac{\Delta^2}{\mu^2} \right) \ , \tag{3}$$

where $\Delta = \Delta_3$. To date the theory has been discussed expanding the dimensionless function $\hat{\mathcal{L}}$ to include all terms of order (∂^2/Δ^2), $(m_s^2/\mu\Delta)^2$, and (M^2/μ^2); the Lagrangian to this order is

$$\begin{aligned}\mathcal{L} \ =\ & \frac{f_\pi^2}{4} \mathrm{Tr} \left(D_0 \Sigma D_0 \Sigma^\dagger - v^2 \mathrm{Tr} \vec{\nabla}\Sigma \cdot \vec{\nabla}\Sigma \right) \\ & + \frac{f_{\eta'}^2}{32} \left(\partial_0 V \partial_0 V^\dagger - v^2 \vec{\nabla} V \cdot \vec{\nabla} V^\dagger \right) \\ & + a_3 V^\dagger \left[(\mathrm{Tr} M\Sigma)^2 - \mathrm{Tr} M\Sigma M\Sigma \right] + \ \mathrm{h.c.}\end{aligned} \tag{4}$$

The covariant derivatives in Eq. (4) contain the Bedaque-Schäfer term [13] that acts as a dynamically generated chemical potential for flavor symmetries,

$$D_0 \Sigma = \partial_0 \Sigma + i \left[M^2/2\mu, \Sigma \right] . \tag{5}$$

By making use of the effective theory introduced by Hong [15], all of the parameters in Eq. (4) have been computed to zeroth order in α_s [10, 11, 16]. The results we will use in this paper are

$$f_\pi^2 = \frac{21 - 8\ln 2}{18} \frac{\mu^2}{2\pi^2} \ , \qquad a_3 = \frac{3}{4\pi^2} \left(\Delta_3\right)^2 \ . \tag{6}$$

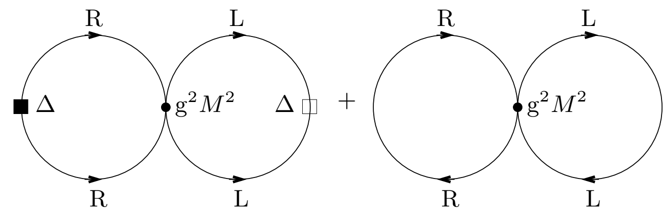

FIG. 1: Feynman diagrams corresponding to the g^2M^2 terms in the effective potential. The open and solid squares denote insertions of the gap, and the dot is an effective vertex which describes chirality changing quark-quark scattering amplitudes.

The a_3 term in Eq. (4), which gives rise to the meson masses, can be rewritten as $2a_3 V \,\mathrm{Tr}\tilde{M}\Sigma + \mathrm{h.c.}$ where $\tilde{M} = \mathrm{diag}(mm_s, mm_s, m^2)$. It is therefore apparent that there exists a meson with square mass proportional to $m^2\Delta^2/\mu^2$, corresponding to fluctuation in the $\{3,3\}$ component of $V\Sigma$, which is exceedingly light. A major point of this paper is that it does not make sense to keep a term proportional to m^2 while dropping ones proportional to $\alpha_s m_s^2$, given that $(m/m_s)^2 \simeq 1 \times 10^{-3}$. There are various small parameters that control the chiral expansion in the CFL phase. Defining $\left(m_s^2/\mu\Delta_3\right)^2 \equiv \delta$ and $m^2/\mu^2 \equiv \epsilon$, we take $m_s m/\mu^2 \sim \delta$ and $\alpha_s m_s^2/\mu^2 \sim \epsilon$ with $\delta^2 \ll \epsilon \ll \delta$, expanding the Lagrangian to $O(\delta)$ and $O(\epsilon)$.

Fig. 1 shows the leading order graphs that contribute to the $O(M^2)$ terms in the chiral effective lagrangian. Fermion lines in the graph on the left correspond to the anomalous quasi-particle propagators in the CFL phase and depend on the gaps $\Delta_{3,6}$. The four-fermion vertex corresponds to a chirality changing four-fermion interaction of order $O(g^2M^2)$ [16]. The Feynman diagram on the left in Fig. 1 gives rise to the a_3 term in in the low energy Lagrangian Eq. (4), as well as the new term

$$\delta\mathcal{L}_6 \ =\ -a_6 \, V^\dagger \left[(\mathrm{Tr} M\Sigma)^2 + \mathrm{Tr} M\Sigma M\Sigma \right] + \mathrm{h.c.} \ , \tag{7}$$

$$a_6 \ =\ \frac{3}{8\pi^2} \left(\Delta_6\right)^2 , \tag{8}$$

which is proportional to α_s, given the relation Eq. (2) [17]. Perturbative corrections to the Feynman diagram in Fig. 1 that are not summed by the gap equation contribute at $O(\alpha_s m m_s \Delta^2)$. We neglect these terms in this work. We also neglect the contribution from the graph on the right of Fig. 1 which is of order $O(m_s^2\Delta^4/\mu^2)$, which is exponentially small as compared to the terms we have kept.

The contributions to the vacuum energy proportional to a_3 and a_6 are opposite in sign, reflecting the fact that one gluon exchange is attractive in the color triplet channel, and repulsive in the sextet channel. Because of the minus sign in $\delta\mathcal{L}_6$ one finds that the $O(\alpha_s m_s^2)$ contributions to the square of the meson masses are negative, and can alter the vacuum alignment.

To analyze the vacuum alignment, we consider free energy density of the homogeneous medium arising from

$\mathcal{L}+\delta\mathcal{L}$:

$$\begin{aligned}\Omega &= -\frac{f_\pi^2}{4}\mathrm{Tr}\left|\left[M^2/2\mu,\Sigma\right]\right|^2 \\ &\quad -2(a_3+a_6)V\,\mathrm{Tr}\tilde{M}\Sigma+\mathrm{h.c.} \\ &\quad +2a_6V^\dagger\left(\mathrm{Tr}M\Sigma\right)^2+\mathrm{h.c.}\end{aligned} \tag{9}$$

Neglecting the new a_6 contributions, one finds that Ω is minimized by the vacuum configuration

$$V=1\ ,\quad \Sigma=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & i\sin\theta\\ 0 & i\sin\theta & \cos\theta\end{pmatrix}\equiv\Sigma_K\ , \tag{10}$$

where the angle θ is satisfies

$$\cos\theta=\min\left[1,c_0\right]\ ,\quad c_0\equiv\frac{16ma_3}{m_s^3}\left(\frac{\mu}{f_\pi}\right)^2\ , \tag{11}$$

where we have dropped $O(m/m_s)$ corrections to c_0. This is the kaon condensed phase discussed in [12, 13, 14]. Note that if we neglect the a_6 contribution, then since $\tilde{M}_{33}=m^2$ there exists a very light excitation φ about Σ_K, corresponding to

$$V\Sigma=\begin{pmatrix}1 & & \\ & 1 & \\ & & e^{i\varphi}\end{pmatrix}\Sigma_K\ . \tag{12}$$

or

$$V=e^{i\varphi/3}\ ,\quad \Sigma=\begin{pmatrix}e^{-i\varphi/3} & & \\ & e^{-i\varphi/3} & \\ & & e^{2i\varphi/3}\end{pmatrix}\Sigma_K\ . \tag{13}$$

The φ excitation has a mass $M_\varphi^2\propto(a_3m^2/\mu^2)$ in the absence of kaon condensation, which is very small. Therefore it is in this direction that we expect an instability to arise when the a_6 operator is included, which contributes $\delta M_\varphi^2\sim-(a_6m_s^2/\mu^2)$. With the ansatz Eq. (13), we can expand the free energy to first order in α_s and m/m_s, with $\Omega=\Omega_0+\Omega_1+\ldots$, where

$$\begin{aligned}\Omega_0 &= -\frac{f_\pi^2m_s^4\sin^2\theta}{8\mu^2}-4mm_sa_3\cos\theta \\ \Omega_1 &= -4\left(a_3m^2\cos\theta-a_6m_s^2\cos^2\theta\right)\cos\varphi\ .\end{aligned} \tag{14}$$

We are neglecting contributions to the free energy of order mm_sa_6, $m^2m_s^2$ and smaller.

Minimizing Ω_0 with respect to θ determines the K^0 condensate. The subleading contribution to the free energy, Ω_1, has a negligible effect on the kaon condensate, which is still given by equ. (11), but is the leading contribution to the potential for the angle φ. Minimizing with respect to φ yields

$$\cos\varphi=\mathrm{Sign}\left[a_3m^2-a_6m_s^2\cos\theta\right]\ . \tag{15}$$

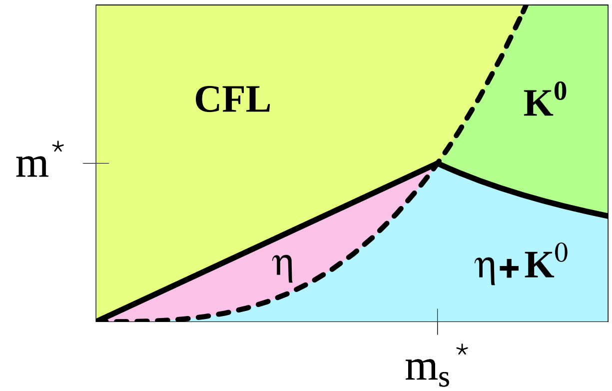

FIG. 2: The phase diagram as a function of light quark mass m and strange quark mass m_s. The phases marked CFL, K^0, η, and $\eta+K^0$ are respectively the CFL phase without meson condensation, with kaon condensation, with η condensation, and with both η and K^0 condensation. Phase transitions are represented by a solid line if first order, a dashed line if second order. The location of the tetracritical point is $\{m_s^*,m^*\}$, given in Eq. (16); including subleading effects causes it to split into two tricritical points.

For brevity we will refer to the phase with $\cos\varphi=-1$ as an "η" condensate, although the mode that condenses is actually a linear combination of η and η'. (Note that in this phase $V=e^{i\pi/3}$; since V has $U(1)_A$ charge equal to four, this phase is equivalent to a strong CP angle $\bar\theta_{QCD}=\pi/2$.) Thus we see that we have four different phases, depending on the quark masses, corresponding to whether or not the kaon and/or the η condense.

The symmetry structure of the four phases is as follows. The ordinary CFL phase (neglecting isospin breaking) possesses an $H=SU(2)\times U(1)_Y\times P$ symmetry, where P is parity, under which $\Sigma\to\Sigma^\dagger$ and $V\to V^\dagger$. Neutral kaon condensation by itself breaks H down to $U(1)_{\mathrm{em}}\times P'$, where P' is a combination of ordinary parity and a discrete hypercharge rotation $\exp(i\pi\hat{Y})$, and $U(1)_{\mathrm{em}}$ is the electromagnetic group. The η condensate by itself breaks the discrete parity symmetry P, but no continuous symmetries [18]. The phase with both η and K^0 condensation breaks H to $U(1)_{em}$.

The phase diagram is easily constructed by means of Eqs. (11,15). The phase boundary for kaon condensation is second order and is described by an equation of the form $m\propto m_s^3$; it is unaffected by the existence or absence of η condensation at the order in α_s and m/m_s that we are working. The phase boundary for η condensation is first order, as one might expect for the breaking of a discrete symmetry. However, to the order we work the φ angle does not encounter a free energy barrier at the phase transition, and so it is possible that higher order corrections could render the phase transition second order. It is described by a line $m\propto m_s$ in the region without kaon condensation, and the curve $m\propto 1/m_s$ in the kaon condensed region. The phase diagram shown in Fig. 2. At this order there is a tetra-critical point, which

is separated into two tricritical points when higher order corrections are included. The coordinates of this critical point are (m_s^*, m^*) given by

$$m^* = \frac{2(\ln 2)^{3/2}}{9\sqrt{3}\pi}\frac{\mu}{f_\pi}\Delta_3\left(\frac{\alpha_s}{4\pi}\right)^{3/4}$$
$$m_s^* = \frac{2(\ln 2)^{1/2}}{\sqrt{3}\pi}\frac{\mu}{f_\pi}\Delta_3\left(\frac{\alpha_s}{4\pi}\right)^{1/4} \quad (16)$$

If the baryon chemical potential is very large we can use perturbation theory in order to estimate the location of the tetracritical point. If we assume that there are only three flavors of quark, and that $\alpha_s(m_\tau) = 0.353$ as in the real world, then at $\mu = 10^{10}$ GeV we have $\alpha_s/\pi = 0.009$, and the critical values

$$\mu = 10^{10}\,\mathrm{GeV}: \quad m^* = 320\,\mathrm{keV} \quad m_s^* = 89\,\mathrm{MeV}\ . \quad (17)$$

In the regime $\mu \simeq 0.5$ GeV, relevant to the physics of neutron stars, the coupling is large and perturbation theory is not reliable. If we assume that $\alpha_s/\pi \simeq 1$, $\Delta \simeq 100$ MeV, and $f_\pi \simeq 90$ MeV we find

$$\mu = 500\,\mathrm{MeV}: \quad m^* \simeq 4.6\,\mathrm{MeV} \quad m_s^* \simeq 120\,\mathrm{MeV}\ . \quad (18)$$

It is interesting to note that the values of the critical masses are remarkably close to the physical values, particularly in the strong coupling example. As a consequence, we expect that the region of the phase diagram explored in this work and displayed in Fig. 2 may well be relevant to the structure of CFL matter in neutron stars.

We emphasize that if CFL quark matter is the true ground state of quark matter at large baryon density then our study provides a rigorous analysis of the phase diagram for small quark masses in the regime $m < m_s$. As such, it provides the correct starting point for an exploration of the phase structure for larger values of the quark masses, and an important guide for any attempt to numerically simulate QCD in the regime of very large baryon density and low temperature.

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[17] The power of g^2 in the vertex of Fig. 1 is canceled by the infrared logarithm $(\ln\Delta/\mu)^2 \propto 1/g^2$.
[18] If $\langle\bar{q}_L\bar{q}_L q_R q_R\rangle \sim V\Sigma$ were the only order parameter, then the η condensed phase with $\phi = \pi$ and $\theta = 0$ would not break parity, since $V\Sigma$ would equal its hermitean conjugate (see Eq. (13)). However, it is possible to construct a twelve fermion order parameter $\langle\left(q_L^4 q_R^2\right)\left(\bar{q}_L^2\bar{q}_R^4\right)\rangle$ which transforms as $(1,1)_{(0,4)} \sim V$. Therefore $V = e^{i\pi/3}$ serves as an order parameter for parity violation in the η condensed phase.